***Differentiation***

**Introduction:**The derivative is a mathematical operator, which measures the rate of change of a quantity relative to another quantity.The process of finding a derivative is called differentiation. There are many phenomena related changing quantities such as speed of a particle, inflation of currency, intensity of an earthquake and voltage of an electrical signal etc. in the world. In this chapter we will discuss about various techniques of derivative.

**Outcomes:** After successful completion of the chapter, the students will be able to:

1. determine the speed, velocity and acceleration of a particle with respect to time.
2. calculate the rate at which the number of bacteria , the population changes with time.
3. measurethe rate at which the length of a metal rod changes with temperature.
4. find out the rate at which production cost changes with the quantity of a product .

**Increment**:Let be a function of. Let be an increment in the value of  and be the corresponding increment in the value of so that







Here is called the increment ratio.

***Differentiability of a function*:** The derivative of  with respect to *x*( for a particular value of *x*) is denoted by or and defined as,





Provided this limit exists. This is called first principle formula for derivative.

***Existence of Derivative:*** A function is called differentiable at if the left hand derivative and right hand derivative at this pointi.e,



and 

are both exist and equal.

**Problem 01:** A function  is defined as follows:



Discuss the differentiability at and.

**Solution:** Given that,



**1st Part:**For,



Since *R.H.D* does not exist. So the function is not differentiable at .

**2nd Part:**For ,



Since  does not exist. So the function is not differentiable at .

**Problem 02:** A function  is defined as follows:



Discuss the differentiability at and.

**Solution:** Given that,



**1st Part:**For,



Since  does not exist. So the function is not differentiable at .

**2nd Part:**For ,



Since exists. So the function is differentiable at .

**HOMEWORK:**

**Problem 01:** A function  is defined as follows:



Discuss the differentiability at .

**Problem 02:**Discuss the differentiability of the functionat the point and 

**Problem 03:** A function  is defined as follows:



Discuss the differentiability at and.

**Problem 04:** A function  is defined as follows:



Discuss the differentiability at and.

**Problem 05:** A function  is defined as follows:



Discuss the differentiability at .

**Problem-06**: Find the derivative of by first principle formula.

**Solution:**  We have 



By first principle formula we can write,













.

**Problem-07**: Find the derivative of by first principle formula.

**Solution:**  We have 



By first principle formula we can write,





















.

**Problem-08**: Find the derivative of by first principle formula.

**Solution:**  We have 



By first principle formula we can write,

















.

**Problem-09**: Find the derivative of by first principle formula.

**Solution:** we have 



By first principle formula we can write,















.

**Problem-10**: Find the derivative of by first principle formula.

**Solution:**We have 



By first principle formula we can write,

















**Problem-11**: Find the derivative of by first principle formula.

**Solution:** We have 



By first principle formula we can write,















**Problem-12**: Find the derivative of by first principle formula.

**Solution:** We have 



By first principle formula we can write,









**Problem-13**: Find the derivative of by first principle formula.

**Solution:**We have 



By first principle formula we can write,







**Problem-14**: Find the derivative of by first principle formula.

**Solution:** We have 



By first principle formula we can write,























.

**Derivatives of elementary functions:**

1. *where c is a constant.* **2.**
2. **4.**
3. **6.**
4. **8.**
5. **10.**
6. **12.**
7. **14.**
8. **16.**
9. **18.**
10. **20.**
11. **22.**

*where u and v are functions of x.*

***Find the differential coefficient*** *(**)****of the following functions:***



**Homework:-**Find  of the following functions:

1. Ans: 
2. Ans: 
3. Ans: 
4. Ans: 
5. Ans: 
6. Ans: 
7. Ans: 
8. Ans: 
9. Ans: 
10. Ans: 

**Logarithmic differentiation:**If we have functions that are composed of products, quotients and powers, to differentiate such functions it would be convenient first to take logarithm of the function and then differentiate. Such a technique is called the logarithmic differentiation.



**Homework:**Find  of the following functions:

1. Ans: 
2. Ans: 
3. Ans: 
4. Ans: 
5. Ans: 
6. Ans: 

**Parametric Equation:**If in the equation of a curve , *x* and *y* are expressed in terms of a third variable known as parameter i.e, then the equations are called a parametric equation.



**Homework:-**

1. Ans: 
2. Ans: 
3. Ans: 
4. Ans: 

**Theorem-01:**Prove that a differentiable function is always continuous but the converse is not always true.

**Proof:**Let the functionbe differentiable at  i.e. exists, so that by the definition of differentiability we have,

exists and finite quantity.

i.e.

Now  







So by the definition of continuity we can say that the function is continuous at the point .

Again conversely, if the function is continuous at a point, then it may not be differentiable at that point. As for example, we will show that the function  is continuous at the point  but it is not differentiable at this point.

**Continuity test:** We have 







Also the functional value at is .

Since so is continuous at .

**Differentiability test:** We have 







Since so is notdifferentiableat .

Hence, a differentiable function is always continuous but the converse is not always true. **(Proved)**

**Problem-01:**If , then show that  is continuous at but not differentiable .

**Solution:**The given function is

**Continuity test:**





Also the functional value at is .

Since so is continuous at .

**Differentiability test:**



Which does not exist but oscillates between -1 and 1 for all values of except .



Which does not exist but oscillates between -1 and 1 for all values of except.

So the function does not differentiable at the point i.e.  does not exist.

Thus the function  is continuous at but not differentiable. **(Showed)**